

Twist-3 fragmentation effects for A_{LT} in light hadron production from proton-proton collisions

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November 17, 2015

Abstract

We compute the contribution from the twist-3 fragmentation function for light hadron production in collisions between transversely and longitudinally polarized protons, i.e., $p^\uparrow \vec{p} \rightarrow h X$, which can cause a double-spin asymmetry (DSA) A_{LT} . This is a naïve T-even twist-3 observable that we analyze in collinear factorization using both Feynman gauge and lightcone gauge as well as give a general proof of color gauge invariance. So far only twist-3 effects in the transversely polarized proton have been studied for A_{LT} in $p^\uparrow \vec{p} \rightarrow h X$. However, there are indications that the naïve T-odd transverse single-spin asymmetry (SSA) A_N in $p^\uparrow p \rightarrow h X$ is dominated not by such distribution effects but rather by a fragmentation mechanism. Therefore, one may expect similarly that the fragmentation contribution is important for A_{LT} . Given possible plans at RHIC to measure this observable, it is timely to provide a calculation of this term.

Keywords: twist-3 effects; spin asymmetries; fragmentation

1 Introduction

Spin asymmetries in various hard processes have brought a new perspective to high-energy perturbative QCD theory and phenomenology. Transverse single-spin asymmetries (SSAs) A_N , which are naïve T-odd observables, were first explored in the mid-1970s in $p^\uparrow p \rightarrow \pi X$ at Argonne National Lab [1] and $p Be \rightarrow \Lambda^\uparrow X$ at FermiLab [2]. Both of these measurements led to strikingly large effects that were unexplainable in the naïve parton model [3]. Experiments continued at FermiLab in the 1990s for $p^\uparrow p \rightarrow \pi X$ [4] and most recently at AGS [5, 6] and RHIC [7–14] for $p^\uparrow p \rightarrow \{\pi, K, jet\} X$. All of their measurements likewise produced substantial transverse SSAs. On the theoretical side, it was realized in the 1980s by Efremov and Teryaev that if one went beyond the simple parton model and included (collinear twist-3) quark-gluon-quark correlations in the nucleon, then there was the potential to generate these large effects [15]. A systematic approach was then developed by Qiu and Sterman in the 1990s that presented the collinear twist-3 factorization framework [16–18] with the expectation that one would be able describe transverse SSAs within this perturbative approach. Later a solid foundation was given to this formalism in [19, 20] that proved the cancelation among gauge-noninvariant terms and led to an expression for the twist-3 cross section in terms of the complete set of the twist-3 quark-gluon-quark correlation functions. Over the last decade, several other analyses, including those for the extension to twist-3 fragmentation functions [21–24] and three-gluon correlation functions [25], furthered the progress of this formalism — see also [26–31] and references therein.

For many years the main assumption was that these transverse SSAs were due to effects inside the transversely polarized proton, in particular those embodied by the so-called Qiu-Sterman function T_F [16–18, 26]. However, a fit of the QS function to A_N data led to a result that was inconsistent with an extraction of the Sivers function f_{1T}^\perp [32] from SIDIS, which has a model-independent relation to T_F [33], and became known as the “sign mismatch” crisis [34]. An attempt to resolve this issue through more flexible parameterizations of the Sivers function proved unsuccessful [35], and, by looking at A_N data on the target transverse SSA in inclusive DIS [36, 37], it was argued in fact that the QS function could not be the main cause of A_N [38]. This led to a recent work that examined the impact of fragmentation effects from the outgoing hadron [39] based on the analytical calculation in Ref. [23]. It was determined that this fragmentation term could be the dominant source of A_N in $p^\uparrow p \rightarrow \pi X$ [39].

In addition to A_N , there is another twist-3 observable in proton-proton collisions that can give insight into quark-gluon-quark correlations in the incoming protons and/or outgoing hadron. This is the longitudinal-transverse double-spin asymmetry (DSA) A_{LT} , which, unlike A_N , is a naïve T-even process. The classic reaction for which this effect has been analyzed is A_{LT} in inclusive DIS (see [40] for recent experimental results on this observable). This asymmetry has also been studied in the Drell-Yan process involving two incoming polarized hadrons [41–44]; in inclusive lepton production from W -boson decay in proton-proton scattering [45]; for jet production [46] and pion production [47] in lepton-nucleon collisions; and for direct photon production [48], jet/pion production [49], and D -meson production [50] in proton-proton collisions.

Of these works on A_{LT} , only in Ref. [47] for $\bar{\ell} p^\uparrow \rightarrow \pi X$ was the twist-3 fragmentation piece calculated (we will see the structure of that result persists in our computation), whereas the fragmentation term for $p^\uparrow \bar{p} \rightarrow \pi X$ has never been studied. Like with A_N , there is no reason *a priori* that this piece cannot be important or perhaps dominant in the asymmetry. Given the possible plans by the PHENIX Collaboration at RHIC to measure A_{LT} for pions [51]¹, we feel a calculation of the fragmentation term for this final state is needed at this time. Furthermore, like prior research in the literature [21–24, 47, 54, 55], this work will continue to establish/verify the theoretical techniques used in collinear twist-3 fragmentation calculations. The remainder of the paper is organized as follows: in Sec. 2 we introduce the twist-3 fragmentation functions relevant for spin-0 hadron production. Next, in Sec. 3 we discuss the calculation of the polarized cross section formula for A_{LT} . Finally, in Sec. 4 we summarize our work and give an outlook. A general proof of the color gauge invariance of our result is given in the Appendix.

2 Twist-3 fragmentation functions for spin-0 hadrons

We now define the set of twist-3 fragmentation functions relevant for spin-0 hadron production. The quark-quark matrix element gives two purely real twist-3 functions, which read

$$\begin{aligned} \frac{1}{N} \sum_X \int \frac{d\lambda}{2\pi} e^{-i\frac{\lambda}{z}} \langle 0 | \psi_i^q(0) | P_h; X \rangle \langle P_h; X | \bar{\psi}_j^q(\lambda w) | 0 \rangle \\ = \frac{M_\chi}{z} (\mathbf{1})_{ij} \hat{e}_1^{h/q}(z) + \frac{M_\chi}{2z} (\sigma_{\lambda\alpha} i\gamma_5)_{ij} \epsilon^{\lambda\alpha w P_h} \hat{e}_1^{h/q}(z) + \dots, \end{aligned} \quad (1)$$

where ψ_i is a quark field with spinor index i , and we use the simplified notation $\epsilon^{\lambda\alpha w P_h} \equiv \epsilon^{\lambda\alpha\rho\sigma} w_\rho P_{h\sigma}$ (with $\epsilon_{0123} = +1$). The color indices are summed over and divided by the number of colors $N = 3$. The scale M_χ is used to make the functions dimensionless and is on the order of the nucleon mass.²

¹We mention that a clear A_{LT} asymmetry has already been seen by the Hall A Collaboration at Jefferson Lab in SIDIS [52] and $\bar{\ell} n^\uparrow \rightarrow \pi X$ [53].

² M_χ is the scale of nonperturbative chiral-symmetry breaking (CSB), which is said to be on the order of the nucleon mass (~ 1 GeV). We have chosen this scale instead of the light hadron mass M_h since twist-3 functions representing

The vector w^μ is light-like ($w^2 = 0$) and satisfies $P_h \cdot w = 1$. We will suppress the gauge-link operators throughout for simplicity.

Next, we introduce the so-called F -type quark-gluon-quark twist-3 fragmentation functions. We can define two independent functions as

$$\begin{aligned} \frac{1}{N} \sum_X \int \frac{d\lambda}{2\pi} \int \frac{d\mu}{2\pi} e^{-i\frac{\lambda}{z_1}} e^{-i\mu(\frac{1}{z} - \frac{1}{z_1})} \langle 0 | \psi_i^q(0) | P_h; X \rangle \langle P_h; X | \bar{\psi}_j^q(\lambda w) g F^{\alpha w}(\mu w) | 0 \rangle \\ = \frac{M_X}{2z} (\gamma_5 \not{P}_h \gamma_\lambda)_{ij} \epsilon^{\lambda\alpha w P_h} \hat{E}_F^{h/q}(z_1, z) + \dots, \end{aligned} \quad (2)$$

$$\begin{aligned} \frac{1}{N} \sum_X \int \frac{d\lambda}{2\pi} \int \frac{d\mu}{2\pi} e^{-i\frac{\lambda}{z_1}} e^{-i\mu(\frac{1}{z} - \frac{1}{z_1})} \langle 0 | \bar{\psi}_j^q(\lambda w) \psi_i^q(0) | P_h; X \rangle \langle P_h; X | g F^{\alpha w}(\mu w) | 0 \rangle \\ = \frac{M_X}{2z} (\gamma_5 \not{P}_h \gamma_\lambda)_{ij} \epsilon^{\lambda\alpha w P_h} \tilde{E}_F^{h/q}(z_1, z) + \dots, \end{aligned} \quad (3)$$

where $F^{\alpha w}(\mu w)$ is the gluon field strength tensor. We note that both $\hat{E}_F(z_1, z)$ and $\tilde{E}_F(z_1, z)$ in general are complex functions. The correlator $\hat{E}_F(z_1, z)$ has support on $1 > z > 0$ and $z_1 > z$, while $\tilde{E}_F(z_1, z)$ has support on $\frac{1}{z} - \frac{1}{z_1} > 1$, $\frac{1}{z_1} < 0$, and $\frac{1}{z} > 0$ [24, 56].

We can consider the so-called D -type twist-3 fragmentation functions $\hat{E}_D(z_1, z)$ by replacing $g F^{\alpha w}(\mu w)$ in (2) with a covariant derivative $D^\alpha(\mu w) = \partial^\alpha - ig A^\alpha(\mu w)$. However, $\hat{E}_F(z_1, z)$ and $\hat{E}_D(z_1, z)$ can be related through the identity,

$$\hat{E}_D^{h/q}(z_1, z) = P\left(\frac{1}{1/z_1 - 1/z}\right) \hat{E}_F^{h/q}(z_1, z) + \delta\left(\frac{1}{z_1} - \frac{1}{z}\right) \tilde{e}^{h/q}(z), \quad (4)$$

where $\tilde{e}(z)$ is another twist-3 fragmentation function that is pure imaginary and defined as

$$\begin{aligned} \frac{1}{N} \sum_X \int \frac{d\lambda}{2\pi} e^{-i\frac{\lambda}{z}} \langle 0 | [\infty w, 0] \psi_i^q(0) | P_h; X \rangle \langle P_h; X | \bar{\psi}_j^q(\lambda w) [\lambda w, \infty w] | 0 \rangle \overleftarrow{\partial}^\alpha \\ = \frac{M_X}{2z} (\gamma_5 \not{P}_h \gamma_\lambda)_{ij} \epsilon^{\lambda\alpha w P_h} \tilde{e}^{h/q}(z) + \dots. \end{aligned} \quad (5)$$

Note that we have restored the gauge links $[a, b]$ in order to emphasize that $\overleftarrow{\partial}^\alpha$ acts on the λ dependence in both the quark field $\bar{\psi}_j^q(\lambda w)$ and the gauge link $[\lambda w, \infty w]$. The D -type function $\hat{E}_D(z_1, z)$ has another relation associated with the QCD equation of motion,

$$z \int_z^\infty \frac{dz_1}{z_1^2} \hat{E}_D^{h/q}(z_1, z) = \hat{e}_1^{h/q}(z) + i \hat{e}_1^{h/q}(z). \quad (6)$$

By combining Eqs. (4), (6) we can eliminate the D -type function and obtain

$$z \int_z^\infty \frac{dz_1}{z_1^2} P\left(\frac{1}{1/z_1 - 1/z}\right) \hat{E}_F^{h/q}(z_1, z) + z \tilde{e}^{h/q}(z) = \hat{e}_1^{h/q}(z) + i \hat{e}_1^{h/q}(z). \quad (7)$$

The real and imaginary parts of the above relation respectively give

$$z \int_z^\infty \frac{dz_1}{z_1^2} P\left(\frac{1}{1/z_1 - 1/z}\right) \hat{E}_F^{h/q, \Re}(z_1, z) = \hat{e}_1^{h/q}(z), \quad (8)$$

helicity flip effects are due to nonperturbative CSB whereas M_h for the pseudoscalar mesons represents the explicit CSB due to the quark mass.

$$z \int_z^\infty \frac{dz_1}{z_1^2} P\left(\frac{1}{1/z_1 - 1/z}\right) \hat{E}_F^{h/q, \Im}(z_1, z) + z \tilde{e}^{h/q, \Im}(z) = \hat{e}_1^{h/q}(z), \quad (9)$$

where \Re (\Im) indicates the real (imaginary) part of the function. It was shown that Eq. (9) ensures the gauge invariance of the polarized cross section formula in the case of the transverse SSA in SIDIS [24]. We will show that Eq. (8) plays the same role in the case of the longitudinal-transverse DSA in proton-proton collisions.

3 Calculation of the polarized cross section for A_{LT}

We consider the polarized cross section for the production of a light (spin-0) hadron from the collision between a transversely polarized proton and a longitudinally polarized proton,

$$p(P, S_\perp) + p(P', \Lambda) \rightarrow h(P_h) + X, \quad (10)$$

where the momenta and polarizations of the particles are given. The first non-vanishing contribution to the cross section reads

$$\begin{aligned} d\sigma(P_{h\perp}, S_\perp, \Lambda) = & H \otimes f_{a/A(3)} \otimes f_{b/B(2)} \otimes D_{C/c(2)} \\ & + H' \otimes f_{a/A(2)} \otimes f_{b/B(3)} \otimes D_{C/c(2)} \\ & + H'' \otimes f_{a/A(2)} \otimes f_{b/B(2)} \otimes D_{C/c(3)}, \end{aligned} \quad (11)$$

where a sum over partonic channels and parton flavors in each channel is understood. The labels on the functions indicate the parton/proton (or hadron/parton) species and the twist (e.g., $f_{a/A(3)}$ denotes a twist-3 correlator associated with parton a in proton A). These functions are convoluted with hard factors H, H', H'' , which are different for each term. The first term in (11) was already calculated in Ref. [49]. With regards to the second term, for the case of the transverse SSA A_N (where B is now unpolarized), which involves chiral-odd twist-3 unpolarized distributions, the authors of Ref. [57] demonstrated this part is negligible because of the smallness of the hard scattering coefficients. However, in Ref. [43] the authors found in Drell-Yan for A_{LT} this second term, which involves the chiral-odd twist-3 longitudinally polarized distribution, can be as large as the first (chiral-even) term. Therefore, this second term should be analyzed in the future in order to have a complete result for this observable. We will now compute the third term, which involves the chiral-odd twist-3 fragmentation functions introduced in Sec. 2 coupled to the (chiral-odd) transversity function $h_1(x)$ and the helicity distribution $g_1(x)$, both defined in the standard way [41, 58, 59]:

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P, S | \bar{\psi}_j^q(0) \psi_i^q(\lambda n) | P, S \rangle = \frac{1}{2} [(\not{P})_{ij} f_1^q(x) + \Lambda(\gamma^5 \not{P})_{ij} g_1^q(x) + (\gamma_5 \not{S}_\perp \not{P})_{ij} h_1^q(x)], \quad (12)$$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P, S | F^{\alpha n}(0) F^{\beta n}(\lambda n) | P, S \rangle = -\frac{x}{2} [g_\perp^{\alpha\beta} f_1^g(x) + \Lambda i \epsilon^{\alpha\beta P n} g_1^g(x)], \quad (13)$$

where the unpolarized distribution $f_1(x)$ has been included for completeness. The vector n^μ is light-like and satisfies $P \cdot n = 1$.

The techniques for calculating the complete fragmentation term in the collinear twist-3 framework have been laid out in Refs. [23, 24, 55]. In particular, the work in [23] can be used for deriving the result in lightcone gauge, whereas that in [24] can be employed if one chooses Feynman gauge. For the former, one can make straightforward changes to the calculation in Ref. [23] in order to obtain the result for $p^\uparrow \bar{p} \rightarrow h X$ (since a similar process ($p^\uparrow p \rightarrow h X$) was computed there). The channels that one must consider are $qg \rightarrow qg$, $qq' \rightarrow qq'$, $qq \rightarrow qq$, $q\bar{q} \rightarrow q\bar{q}$, $q\bar{q}' \rightarrow q\bar{q}'$, and $\bar{q}q \rightarrow q\bar{q}$ (see Figs. 1–3)

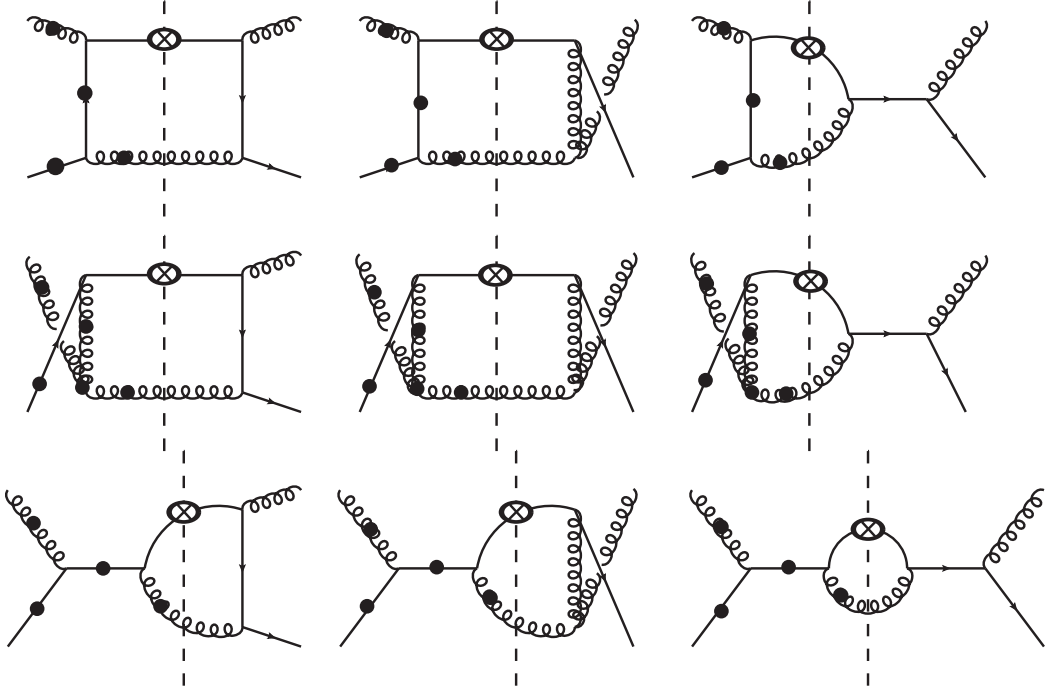


Figure 1: Feynman diagrams for the $qq \rightarrow qq$ channel. The circled cross indicates the parton that fragments. The 9 graphs (ignoring the dots) lead to the hard factor for $\hat{e}_1(z)$ while those with the dots, which represent coherent gluon attachments from the parton line to the fragmentation correlator, give the hard part for $\hat{E}_F^{\mathfrak{R}}(z_1, z)$. The Hermitian conjugate (H.c.) graphs are also taken into account.

plus all the antiquark fragmentation channels found through charge conjugating the aforementioned ones. We note that because of the T-even nature of this observable, the structure of our cross section takes on a different form than A_N . For example, one no longer has contributions from $\hat{e}_1(z)$ or $\tilde{e}(z)$; instead, the quark-quark piece only involves $\hat{e}_1(z)$. Likewise, for the quark-gluon-quark part, one receives contributions from $\hat{E}_F^{\mathfrak{R}}(z_1, z)$ instead of $\tilde{E}_F^{\mathfrak{S}}(z_1, z)$. The hard factors for $\tilde{E}_F(z_1, z)$ (see Fig. 4) vanish, as was also found in Ref. [23] with A_N . Furthermore, unlike A_N , it turns out that the quark-gluon-quark hard factors are independent of z_1 . Thus, one can use Eq. (8) to write this piece in terms of $\hat{e}_1(z)$, and the entire cross section then only involves this function. Such a simplification was also noticed in Ref. [47] for $\vec{\ell} p^\uparrow \rightarrow \pi X$. We then find the fragmentation term in the polarized cross section relevant for A_{LT} to be

$$\frac{P_h^0 d\sigma_{LT}^{Frag}}{d^3 \vec{P}_h} = -\frac{2\alpha_s^2 M_X}{S} \Lambda P_{h\perp} \cdot S_\perp \sum_i \sum_{a,b,c} \int_0^1 \frac{dz}{z^4} \hat{e}_1^{h/c}(z) \int_0^1 \frac{dx'}{x'} g_1^b(x') \int_0^1 \frac{dx}{x} h_1^a(x) \hat{\sigma}_i \delta(\hat{s} + \hat{t} + \hat{u}). \quad (14)$$

The hard factors $\hat{\sigma}_i$ are given by

$$\begin{aligned} \hat{\sigma}_{qq \rightarrow qq} &= -\frac{3}{2} \left[\frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2 \hat{u}} - \frac{1}{N^2} \frac{1}{\hat{u}} \right], & \hat{\sigma}_{qq' \rightarrow qq'} &= \left(1 - \frac{1}{N^2} \right) \frac{\hat{s}}{\hat{t}^2}, \\ \hat{\sigma}_{qq \rightarrow q\bar{q}} &= \left(1 - \frac{1}{N^2} \right) \left[\frac{\hat{s}}{\hat{t}^2} + \frac{1}{N} \frac{\hat{s}(\hat{s} - 2\hat{t})}{2\hat{t}^2 \hat{u}} \right], & \hat{\sigma}_{q\bar{q} \rightarrow q\bar{q}} &= \left(1 - \frac{1}{N^2} \right) \left[\frac{\hat{s}}{\hat{t}^2} + \frac{1}{N} \frac{\hat{u} - 2\hat{t}}{2\hat{t}^2} \right], \\ \hat{\sigma}_{q\bar{q}' \rightarrow q\bar{q}'} &= \hat{\sigma}_{qq' \rightarrow qq'}, & \hat{\sigma}_{\bar{q}q \rightarrow q\bar{q}} &= \frac{1}{N} \left(1 - \frac{1}{N^2} \right) \frac{3}{2\hat{u}}. \end{aligned} \quad (15)$$

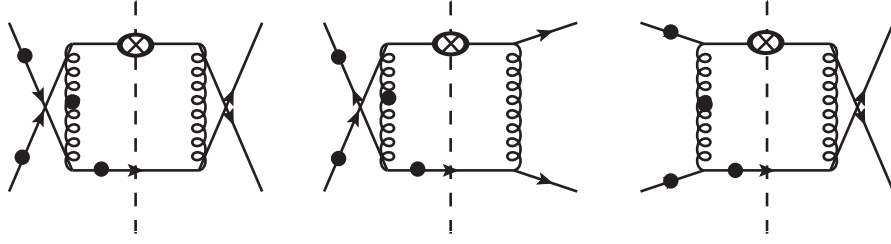


Figure 2: Feynman diagrams for the $qq \rightarrow qq$ channel. The first graph gives the $qq' \rightarrow qq'$ channel.

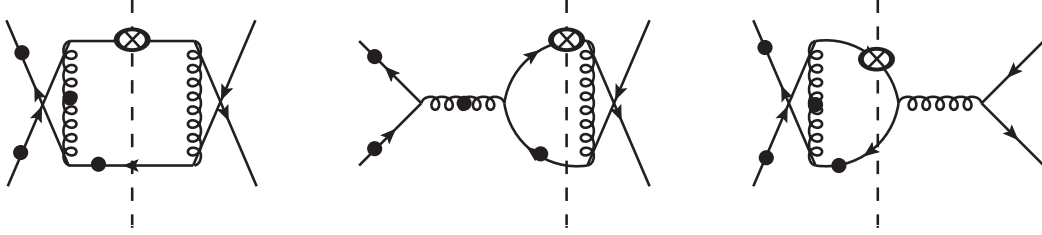


Figure 3: Feynman diagrams for the $q\bar{q} \rightarrow q\bar{q}$ channel. The first graph gives the $q\bar{q}' \rightarrow q\bar{q}'$ channel. Note that the $\bar{q}q \rightarrow q\bar{q}$ channel is found by interchanging the two incoming parton lines.

The Mandelstam variables for the process are defined as $S = (P + P')^2$, $T = (P - P_h)^2$, and $U = (P' - P_h)^2$, which on the partonic level give $\hat{s} = xx'S$, $\hat{t} = xT/z$, and $\hat{u} = x'U/z$. For the antiquark fragmentation channels we find (cf. [23]) $\hat{\sigma}_{\bar{a}\bar{b} \rightarrow \bar{c}\bar{d}} = \hat{\sigma}_{ab \rightarrow cd}$, where $\hat{\sigma}_{ab \rightarrow cd}$ are given in (15). We also calculated the polarized cross section in Feynman gauge using the procedure of Ref. [24] and found agreement with Eqs. (14), (15). A general proof of color gauge invariance can also be found in the Appendix.

4 Summary and outlook

Transverse SSAs A_N in single-inclusive processes (e.g., $p^\uparrow p \rightarrow h X$) are twist-3 observables that have been an intense topic of research for close to 40 years. Large effects have been found that still have an unclear origin. Recently it has been shown that the fragmentation term in collinear twist-3 factorization could be the main cause of A_N in $p^\uparrow p \rightarrow \pi X$ at RHIC [39]. In addition, another twist-3 reaction exists that can also lead to information on quark-gluon-quark correlations in protons/hadrons: the longitudinal-transverse DSA A_{LT} in $p^\uparrow \vec{p} \rightarrow h X$. Already two related observables, A_{LT} in SIDIS [52] and in $\vec{\ell} n^\uparrow \rightarrow \pi X$ [53], have been measured and nonzero effects have been found. However, RHIC, with the only source of (independently manipulated) polarized proton beams in the world, has yet to explore A_{LT} in $p^\uparrow \vec{p} \rightarrow h X$ despite measuring asymmetries for every other combination of proton spins. Just recently, though, the PHENIX Collaboration has put forth plans to make this measurement [51]. Some work has been done previously on A_{LT} that looked at the twist-3 effects in the polarized proton [49]. Motivated by the potential of twist-3 fragmentation effects to dominate A_N [39], and with no reason *a priori* that they should be small, we have computed this term now for A_{LT} . We found that the entire result can be written in terms of a single (twist-3) quark-quark fragmentation function $\hat{e}_1(z)$ and confirmed this using two different gauges. In the future we plan to perform a detailed numerical study of A_{LT} in $p^\uparrow \vec{p} \rightarrow \pi X$ in order to further encourage an experiment at RHIC.

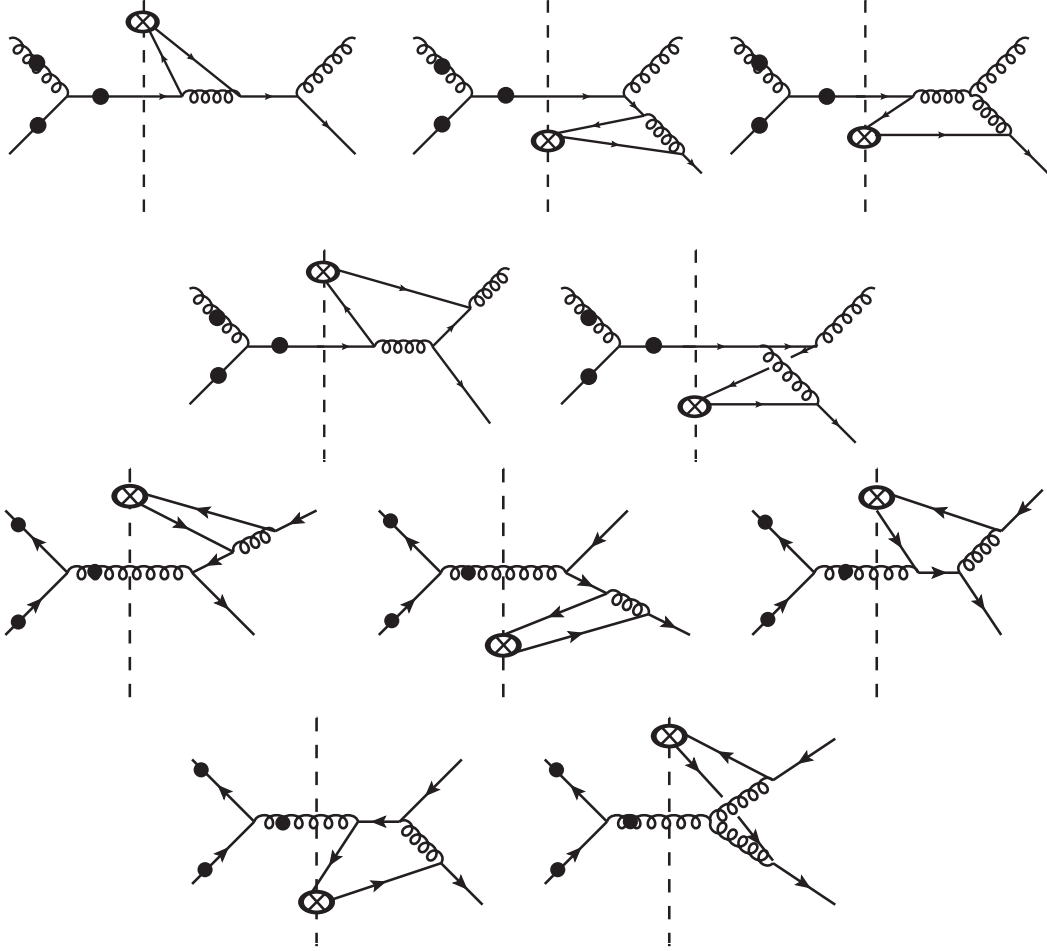


Figure 4: Feynman diagrams for qq and $q\bar{q}$ induced channels where both quarks entering the fragmentation are on the same side of the cut. These lead to the hard factors for $\tilde{E}_F(z_1, z)$. Such graphs cancel after one sums all contributions.

Acknowledgments

This work has been supported by the Grant-in-Aid for Scientific Research from the Japanese Society of Promotion of Science under Contract No. 26287040 (Y.K. and S.Y.) and the RIKEN BNL Research Center (D.P.).

Appendix: General proof of color gauge invariance

We want to show that our calculation of the $qg \rightarrow qg$ channel satisfies the Ward-Takahashi identity (WTI):

$$(xP + x'P' - P_h/z)^\rho d\sigma_{\rho\sigma}^{qg \rightarrow qg} = 0, \quad (16)$$

where $d\sigma_{\rho\sigma}^{qg \rightarrow qg}$ is the partonic hard factor with the polarization tensor for the final unobserved gluon $d^{\rho\sigma}(xP + x'P' - P_h/z)$ removed. The momenta and Lorentz indices are shown in Fig. 5(a). In our computation, we wrote $d\sigma^{qg \rightarrow qg}$ as follows (cf. Fig. 1):

$$d\sigma^{qg \rightarrow qg} = d\sigma^{(a)} + \left(d\sigma^{(b)L} - d\sigma^{(c)L}\right) + \left(d\sigma^{(b)R} - d\sigma^{(c)R}\right), \quad (17)$$

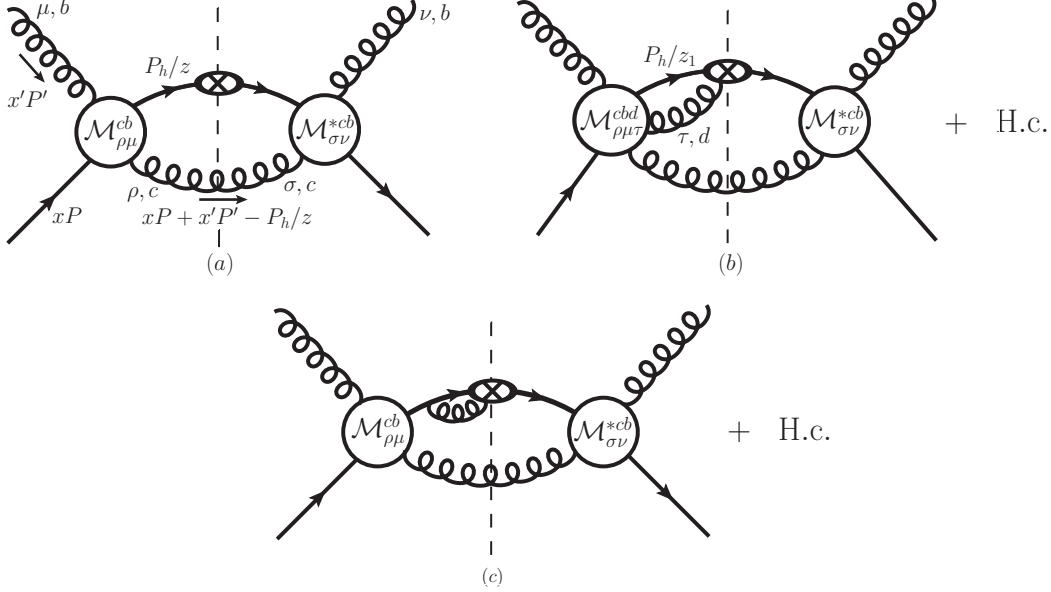


Figure 5: Feynman diagrams that enter our general proof of color gauge invariance: (a) quark-quark and (b), (c) quark-gluon-quark graphs. Note that our calculation does not include in (b) the subset of diagrams given in (c), but rather these topologies are taken care of by those in (a). See the text for details.

where the superscripts correspond to the diagrams in Fig. 5 (L indicates the explicitly shown graph and R its H.c.). One sees immediately that

$$(xP + x'P' - P_h/z)^\rho \left\{ d\sigma_{\rho\sigma}^{(b)L}, d\sigma_{\rho\sigma}^{(b)R}, d\sigma_{\rho\sigma}^{(c)R} \right\} = 0, \quad (18)$$

since from the WTI for $qg \rightarrow qg$ scattering we know

$$(xP + x'P' - P_h/z)^\rho \left\{ \mathcal{M}_{\rho\mu}^{cb}, \mathcal{M}_{\rho\mu\tau}^{cbd} \right\} = 0 \quad (19)$$

if all particles attaching to the respective blob are external (on-shell) lines. The Dirac projections used for the correlators in $d\sigma^{(b)L}$, $d\sigma^{(b)R}$, $d\sigma^{(c)R}$ allow these connecting particles to meet this criteria.³ Therefore, we now have

$$\begin{aligned} (xP + x'P' - P_h/z)^\rho d\sigma_{\rho\sigma}^{qg \rightarrow qg} &= (xP + x'P' - P_h/z)^\rho \left(d\sigma_{\rho\sigma}^{(a)} - d\sigma_{\rho\sigma}^{(c)L} \right) \\ &\sim (xP + x'P' - P_h/z)^\rho \epsilon^{\mu\nu P n} \\ &\quad \times \left\{ -\frac{i}{2} \epsilon^{\tau\tau' P_h w} \text{Tr} \left[\not{\$}_\perp \not{P} \gamma_5 \mathcal{M}_{\sigma\nu}^{*cb} \gamma_{\tau'} (\not{P}_h/z) \gamma_5 \gamma_\tau \frac{\not{P}_h/z}{(P_h/z)^2} \mathcal{M}_{\rho\mu}^{cb} \right] \right. \\ &\quad \left. + \text{Tr} \left[\not{\$}_\perp \not{P} \gamma_5 \mathcal{M}_{\sigma\nu}^{*cb} \mathcal{M}_{\rho\mu}^{cb} \right] \right\}. \end{aligned} \quad (20)$$

One can show that

$$(xP + x'P' - P_h/z)^\rho \epsilon^{\mu\nu P n} \mathcal{M}_{\rho\mu}^{cb} \not{P}$$

³The diagram in Fig. 5(a) does not satisfy this requirement because the fragmentation correlator is projected out with $\mathbf{1}$.

$$\begin{aligned}
&\sim \epsilon^{\mu\nu Pn} (\not{P}_h/z) \left[\frac{i(x\not{P} + x'\not{P}')}{\hat{s}} \gamma_\mu T^c T^b + \frac{P_{h\mu}/z}{\hat{t}} f^{bdc} T^d \right] \not{P} \\
&\equiv (\not{P}_h/z) \mathcal{M}'^{\nu cb} \not{P},
\end{aligned} \tag{21}$$

where f^{abc} are the structure constants and T^a the generators of $SU(3)$. This leads to

$$\begin{aligned}
(xP + x'P' - P_h/z)^\rho d\sigma_{\rho\sigma}^{qg \rightarrow qg} &\sim -\frac{i}{2} \epsilon^{\tau\tau' P_h w} \text{Tr} \left[\not{S}_\perp \not{P} \gamma_5 \mathcal{M}_{\sigma\nu}^{*cb} \gamma_{\tau'} (\not{P}_h/z) \gamma_5 \gamma_\tau \mathcal{M}'^{\nu cb} \right] \\
&+ \text{Tr} \left[\not{S}_\perp \not{P} \gamma_5 \mathcal{M}_{\sigma\nu}^{*cb} (\not{P}_h/z) \mathcal{M}'^{\nu cb} \right].
\end{aligned} \tag{22}$$

We can simplify this expression further through use of the identities

$$\gamma_5 \sigma_{\mu\nu} = -\frac{i}{2} \epsilon_{\mu\nu}^{\alpha\beta} \sigma_{\alpha\beta}, \quad \epsilon^{\mu\nu\rho\sigma} \epsilon_{\mu\nu}^{\alpha\beta} = -2(g^{\rho\alpha} g^{\sigma\beta} - g^{\rho\beta} g^{\sigma\alpha}) \tag{23}$$

and obtain

$$\begin{aligned}
(xP + x'P' - P_h/z)^\rho d\sigma_{\rho\sigma}^{qg \rightarrow qg} &\sim \frac{1}{2} \text{Tr} \left[\not{S}_\perp \not{P} \gamma_5 \mathcal{M}_{\sigma\nu}^{*cb} (\not{P}_h/z) (\not{P}_h \psi - \psi \not{P}_h) \mathcal{M}'^{\nu cb} \right] \\
&+ \text{Tr} \left[\not{S}_\perp \not{P} \gamma_5 \mathcal{M}_{\sigma\nu}^{*cb} (\not{P}_h/z) \mathcal{M}'^{\nu cb} \right] \\
&= 0.
\end{aligned} \tag{24}$$

Thus, the $qg \rightarrow qg$ channel satisfies the WTI.

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